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Technical Note

A closed form solution for temperature rise inside solid substrate due to time exponentially varying pulse

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Abstract

Modeling of laser heating process minimizes the experimental cost and enables to optimize the process parameters for improved end product quality. In the present study, an analytical solution for laser conduction limited heating due to time exponentially varying pulse is presented. The governing equation of heat diffusion is solved analytically using a Laplace transformation method. The closed form solution is validated by a solution for a step input pulse intensity presented in the previous study as well as numerical predictions. Temperature rise inside the substrate material is computed for steel. It is found that the present solution reduces to previous solution once the pulse parameter ($\beta = 0$) are set to zero. Temperatures obtained from the closed form solution agree well with the numerical predictions. Moreover, temperature rises rapidly in the surface vicinity due to time exponentially varying pulse. The pulse parameter (β^*/γ^*) has a significant effect on the temperature rise. In this case, low value of (β^*/γ^*) results in high temperature rise in the surface vicinity of the substrate material. © 2002 Elsevier Science Ltd. All rights reserved.

1. Introduction

Laser conduction limited heating process is important when laser is used as a surface treatment tool in industry, since the quality of the end product strongly depends on the laser heating conditions. The modeling of laser heating process gives insight into the physical parameters affecting the heating process, minimizes the experimental cost, and enables to optimize the process parameters such as laser and workpiece parameters.

Considerable research studies were carried out to explore the laser heating process. Some of them were involved with the analytical solution to the heating problem. Moreover, analytical solution gives explicit functional relation between temperature and laser pulse and workpiece material properties as well as reduces considerably the simulation time. Consequently, analytical solution to the heating process is fruitful for laser heating applications. An analytical solution for a pulse laser heating was introduced by Ready [1]. Laser heating mechanism including evaporation process during laser drilling of the metallic substrates was investigated analytically by Yilbas et al. [2]. They obtained the closed form solution for the temperature rise due to a step input intensity pulse and determined the drilling efficiency. Laser heating of a two-layer system was formulated by Al-Adawi et al. [3] using a Laplace transformation method. The time required for the melting of the films situated on a glass substance was computed. Heat conduction in a moving semi-infinite solid subjected to a pulsed laser irradiation was studied analytically by Modest and Abakians [4]. They obtained a closed form solution for the temperature distribution inside the substrate material. However, their solution was limited to a surface heat source heating where as deposited laser energy was absorbed by the substrate material within the absorption depth. The analytical solution for a two-dimensional solid with a surface heat source was obtained by Aviles-Ramos et al. [5]. The closed form solution enables to predict the surface temperature rise and the heat flux across the surface. An analytical solution for a conduction limited heating was studied by Yilbas and

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Nomenclature

C_p	specific heat (J/kg K)
I_0	laser peak power intensity (W/m^2)
I_1	laser peak power intensity available at the surface $(I_0(1 - r_f))$ (W/m^2)
k	thermal conductivity (W/m K)
r_f	reflection coefficient
t	time (s)
p	Laplace variable (1/s)
x	spatial coordinates (m)
x^*	dimensionless spatial coordinate ($x\delta$)
T	temperature (K)
T^*	dimensionless temperature $((T/I_1)k\delta)$
T_s	surface temperature (K)
t	time (s)
t^*	dimensionless time ($\alpha\delta^2 t$)
α	thermal diffusivity (m^2/s)
β	pulse parameter (1/s)
β^*	dimensionless pulse parameter (βt)
γ	pulse parameter (1/s)
γ^*	dimensionless pulse parameter (γt)
δ	absorption coefficient (1/m)
ρ	density (kg/m^3)

Shuja [6]. They obtained a closed form solution for a temperature distribution inside the solid substrates and introduced a dimensionless equilibrium temperature inside the substrate material.

In practical laser heating applications, the actual laser pulse has time-dependent power intensity distribution. In most of the analytical approaches the temporal variation of laser power intensity distribution was ignored. This, in turn, results in considerable errors in the solution of the heat conduction equation for a time-dependent laser pulse. Therefore, temporal variation of laser power intensity distribution resembling the actual laser pulse needs to incorporated in the analysis when obtaining the closed form solution to the practical pulsed laser heating process. Yilbas [7] obtained the closed form solution for the surface temperature rise due to time exponentially varying laser heating pulse. However, the study was limited to a formulation of the surface temperature rise. Consequently, in the present study, an analytical solution for the intensity time exponentially varying pulse is considered and the closed form solution for the temperature rise inside the substrate material is obtained. The effect of pulse parameters (β and γ) on the resulting temperature distribution is investigated. The closed form solution is validated through reducing the present solution to the solution for a step intensity pulse presented in the previous study [8] as well as comparing the results with their counterparts predicted from the numerical simulations.

2. Mathematical modeling

The Fourier heat transfer equation for a laser time exponentially varying heating pulse can be written as:

$$\frac{\partial^2 T}{\partial x^2} + \frac{I_1 \delta}{k}(\mathrm{e}^{-\beta t} - \mathrm{e}^{-\gamma t})\mathrm{e}^{-\delta x} = \frac{1}{\alpha}\frac{\partial T}{\partial t}. \quad (1)$$

The temporal variation of laser output pulse is not easily fitted by a simple mathematical expression and it may be approximated by a form of exponential function. Consequently, in order to account for the rise and fall times of a laser pulse, two exponential terms resembling almost a practical laser pulse are accommodated in the analysis (Fig. 1) [8], i.e.,

$$I = I_1(\mathrm{e}^{-\beta t} - \mathrm{e}^{-\gamma t}), \quad (2)$$

where

$$I_1 = (1 - r_f)I_0,$$

where r_f is the reflection coefficient and I_0 is the peak power intensity, and parameters β and γ can be chosen to give the appropriate rise time for the pulse. Since the governing equation of heat transfer is linear (Eq. (1)), it is unnecessary to solve Eq. (1) for a complete pulse (full pulse including β and γ terms). Therefore, the complete solution can be obtained by summation of the solutions for the individual parts of the time exponential pulse (half pulse including β or γ term only). It should be noted that the solution of half pulse satisfies the initial and boundary conditions for a full pulse; therefore, the rule of superposition is applicable.

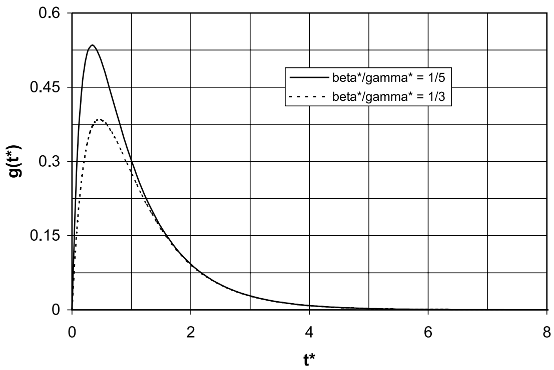

Fig. 1. Temporal variation of exponential function resembling the heating pulse $g(t^*) = \exp(-\beta^*) - \exp(-\gamma^*)$.

Since the equation is linear, the complete solution can be obtained by summation of the solutions for the individual parts of the time exponential. Therefore, the Fourier equation to be solved reduces to:

$$\frac{\partial^2 T}{\partial x^2}+\frac{I_1\delta}{k}\exp[-(\beta t+\delta x)]=\frac{1}{\alpha}\frac{\partial T}{\partial t} \tag{3}$$

with the boundary conditions:

$$\begin{aligned}&\text{At the surface} \Rightarrow x=0\rightarrow\left[\frac{\partial T}{\partial x}\right]_{x=0}=0.\\&\text{At infinity} \Rightarrow x=\infty\rightarrow T(\infty,t)=0.\end{aligned} \tag{4}$$

Initial condition:

$$\text{At time zero} \Rightarrow t=0\rightarrow T(x,0)=0. \tag{5}$$

The solution of Eq. (3) is possible in the Laplace domain and inversion gives the solution. Therefore, the Laplace transform of Eq. (3) and introducing the initial condition yields:

$$\frac{\partial^2 \overline{T}}{\partial x^2}-g^2\overline{T}=-\frac{I_1\delta\exp(-\delta x)}{k(p+\beta)}, \tag{6}$$

where $T=T(x,p)$, $g^2=p/\alpha$, and p is the transform variable. Eq. (6) has the solution:

$$\overline{T}=A\exp(-gx)+B\exp(gx)-\frac{I_1\delta\exp(-\delta x)}{k(p+\beta)(\delta^2-g^2)}, \tag{7}$$

where A and B are constants. Introducing the boundary conditions into Eq. (7), the constants A and B can be found, i.e.:

$$B=0,$$

$$A=\frac{I_1\delta^2}{kg(p+\beta)(\delta^2-g^2)},$$

After substitution of constants A and B into Eq. (7), it yields:

$$\overline{T}=-\frac{I_1\delta}{k(p+\beta)}\left[\frac{\delta\exp(-gx)}{g(g^2-\delta^2)}-\frac{\exp(-\delta x)}{(g^2-\delta^2)}\right]. \tag{8}$$

Eq. (8) gives the solution for temperature in the Laplace domain. In order to obtain the solution in the physical domain, the Laplace inversion of Eq. (8) is necessary. There are two ways to achieve the inversion, since the complete solution is a product of two p-functions. Consequently, in the first method, the inversion of solution can be expressed as a convolution integral which then can be evaluated or in the second method, entailing expansion of the functions into partial fractions. The second method is adopted at present due to the simplicity. Consider the term:

$$\frac{\delta\exp(-gx)}{g(p+\beta)(g^2-\delta^2)},$$

which can be written as

$$\frac{\alpha\delta\exp(-gx)}{g(p+\beta)(p-\alpha\delta^2)}.$$

This can be expanded to:

$$\frac{\alpha\delta\exp(-gx)}{g(\beta+\alpha\delta^2)}\left[\frac{1}{p-\alpha\delta^2}-\frac{1}{p+\beta}\right]. \tag{9}$$

The inversion of above expression gives [10]:

$$\begin{aligned}&\frac{1}{2}\left(\frac{\alpha}{\beta+\alpha\delta^2}\right)\Bigg\{\mathrm{i}\delta\sqrt{\frac{\alpha}{\beta}}\exp(\beta t)\Bigg[\exp\left(-\mathrm{i}x\sqrt{\frac{\beta}{\alpha}}\right)\\&\times\operatorname{erf}c\left(\frac{x}{2\sqrt{\alpha t}}.-\mathrm{i}\sqrt{\beta t}\right)\\&-\exp\left(\mathrm{i}x\sqrt{\frac{\beta}{\alpha}}\right)\operatorname{erf}c\left(\frac{x}{2\sqrt{\alpha t}}+\mathrm{i}\sqrt{\beta t}\right)\Bigg]\\&+\exp(\alpha\delta^2 t)\Bigg[\exp(-\delta x)\operatorname{erf}c\left(\frac{x}{2\sqrt{\alpha t}}-\sqrt{\alpha\delta^2 t}\right)\\&-\exp(\delta x)\operatorname{erf}c\left(\frac{x}{2\sqrt{\alpha t}}+\sqrt{\alpha\delta^2 t}\right)\Bigg]\Bigg\}.\end{aligned} \tag{10}$$

In a similar way, term:

$$\frac{\exp(-\delta x)}{(p+\beta)(g^2-\delta^2)}$$

can be written as:

$$\frac{\alpha\exp(-\delta x)}{(p+\beta)(p-\alpha\delta^2)}. \tag{11}$$

The Laplace inversion of Eq. (11) yields:

$$\frac{\alpha\exp(-\delta x)}{\beta+\alpha\delta^2}[\exp(\alpha\delta^2 t)-\exp(-\beta t)]. \tag{12}$$

Consequently, substituting Eqs. (10) and (12) into Eq. (8) gives the full solution obtained by inverse Laplace transformation, i.e.:

$$\begin{aligned}T(x,t)=\frac{I_1\delta}{2k}\left(\frac{\alpha}{\beta+\alpha\delta^2}\right)\Bigg\{&\mathrm{i}\delta\sqrt{\frac{\alpha}{\beta}}\exp(-\beta t)\\&\times\Bigg[\exp\left(\mathrm{i}x\sqrt{\frac{\beta}{\alpha}}\right)\operatorname{erf}c\left(\frac{x}{2\sqrt{\alpha t}}+\mathrm{i}\sqrt{\beta t}\right)\\&-\exp\left(-\mathrm{i}x\sqrt{\frac{\beta}{\alpha}}\right)\operatorname{erf}c\left(\frac{x}{2\sqrt{\alpha t}}-\mathrm{i}\sqrt{\beta t}\right)\Bigg]\\&+\exp(\alpha\delta^2 t)\Bigg[\exp(\delta x)\operatorname{erf}c\left(\frac{x}{2\sqrt{\alpha t}}+\delta\sqrt{\alpha t}\right)\\&-\exp(-\delta x)\operatorname{erf}c\left(\frac{x}{2\sqrt{\alpha t}}-\delta\sqrt{\alpha t}\right)\Bigg]\\&+2\exp(-\delta x)[\exp(\alpha\delta^2 t)-\exp(-\beta t)].\Bigg\}\end{aligned} \tag{13}$$

After using the relationship:

$$\mathrm{erf}\,c(-z) = 2 - \mathrm{erf}\,c(z)$$

and

$$\mathrm{erf}\,c(z) = 1 - \mathrm{erf}\,(z)$$

gives the closed form solution for the temperature rise inside the substrate material:

$$\begin{aligned} T(x,t) = {} & \frac{I_1\delta}{2k}\left(\frac{\alpha}{\beta+\alpha\delta^2}\right)\left\{ \mathrm{i}\delta\sqrt{\frac{\alpha}{\beta}}\exp(-\beta t) \right. \\ & \times\left[\exp\left(\mathrm{i}x\sqrt{\frac{\beta}{\alpha}}\right)\mathrm{erf}\,c\left(\frac{x}{2\sqrt{\alpha t}}+\mathrm{i}\sqrt{\beta t}\right)\right. \\ & \left. -\exp\left(-\mathrm{i}x\sqrt{\frac{\beta}{\alpha}}\right)\mathrm{erf}\,c\left(\frac{x}{2\sqrt{\alpha t}}-\mathrm{i}\sqrt{\beta t}\right)\right] \\ & +\exp(\alpha\delta^2 t)\left[\exp(\delta x)\mathrm{erf}\,c\left(\frac{x}{2\sqrt{\alpha t}}+\delta\sqrt{\alpha t}\right)\right. \\ & -\exp(-\delta x)\mathrm{erf}\,c\left(\delta\sqrt{\alpha t}-\frac{x}{2\sqrt{\alpha t}}\right) \\ & \left.\left. -2\exp(-(\beta t+\delta x))\right]\right\}. \end{aligned} \tag{14}$$

Eq. (14) can be non-dimensionalized using the following non-dimensional parameters:

$$t^* = \alpha\delta^2 t,$$

$$\beta^* = \beta t,$$

$$x^* = x\delta,$$

$$T^* = \frac{T}{I_1}k\delta.$$

The resulting non-dimensional equation is

$$\begin{aligned} T^*(x^*,t^*) = {} & \frac{1}{2}\left(\frac{t^*}{\beta^*+t^*}\right)\left\{ \mathrm{i}\sqrt{\frac{t^*}{\beta^*}}\exp(-\beta^*) \right. \\ & \times\left[\exp\left(\mathrm{i}x^*\sqrt{\frac{\beta^*}{t^*}}\right)\mathrm{erf}\,c\left(\frac{x^*}{2\sqrt{t^*}}+\mathrm{i}\sqrt{\beta^*}\right)\right. \\ & \left. -\exp\left(-\mathrm{i}x^*\sqrt{\frac{\beta^*}{t^*}}\right)\mathrm{erf}\,c\left(\frac{x^*}{2\sqrt{t^*}}-\mathrm{i}\sqrt{\beta^*}\right)\right] \\ & +\exp(\alpha\delta^2 t)\left[\exp(x^*)\mathrm{erf}\,c\left(\frac{x^*}{2\sqrt{t^*}}+\sqrt{t^*}\right)\right. \\ & -\exp(-x^*)\mathrm{erf}\,c\left(\sqrt{t^*}-\frac{x^*}{2\sqrt{t^*}}\right) \\ & \left.\left. -2\exp(-(\beta^*+x^*))\right]\right\}. \end{aligned} \tag{15}$$

The closed form solution for the full pulse can be written as

$$\begin{aligned} T^*(x^*,t^*) = {} & \frac{1}{2}\left\{\left(\frac{t^*}{\beta^*+t^*}\right)\left(\mathrm{i}\sqrt{\frac{t^*}{\beta^*}}\exp(-\beta^*)\right.\right. \\ & \times\left[\exp\left(\mathrm{i}x^*\sqrt{\frac{\beta^*}{t^*}}\right)\mathrm{erf}\,c\left(\frac{x^*}{2\sqrt{t^*}}+\mathrm{i}\sqrt{\beta^*}\right)\right. \\ & \left. -\exp\left(-\mathrm{i}x^*\sqrt{\frac{\beta^*}{t^*}}\right)\mathrm{erf}\,c\left(\frac{x^*}{2\sqrt{t^*}}-\mathrm{i}\sqrt{\beta^*}\right)\right] \\ & +\exp(t^*)\left[\exp(x^*)\mathrm{erf}\,c\left(\frac{x^*}{2\sqrt{t^*}}+\sqrt{t^*}\right)\right. \\ & -\exp(-x^*)\mathrm{erf}\,c\left(\sqrt{t^*}-\frac{x^*}{2\sqrt{t^*}}\right) \\ & \left.\left. -2\exp(-(\beta^*+x^*))\right]\right) \\ & -\left(\frac{t^*}{\gamma^*+t^*}\right)\left(\mathrm{i}\sqrt{\frac{t^*}{\gamma^*}}\exp(-\gamma^*)\right. \\ & \times\left[\exp\left(\mathrm{i}x^*\sqrt{\frac{\gamma^*}{t^*}}\right)\mathrm{erf}\,c\left(\frac{x^*}{2\sqrt{t^*}}+\mathrm{i}\sqrt{\gamma^*}\right)\right. \\ & \left. -\exp\left(-\mathrm{i}x^*\sqrt{\frac{\gamma^*}{t^*}}\right)\mathrm{erf}\,c\left(\frac{x^*}{2\sqrt{t^*}}-\mathrm{i}\sqrt{\gamma^*}\right)\right] \\ & +\exp(t^*)\left[\exp(x^*)\mathrm{erf}\,c\left(\frac{x^*}{2\sqrt{t^*}}+\sqrt{t^*}\right)\right. \\ & -\exp(-x^*)\mathrm{erf}\,c\left(\sqrt{t^*}-\frac{x^*}{2\sqrt{t^*}}\right) \\ & \left.\left.\left. -2\exp(-(\gamma^*+x^*))\right]\right)\right\}, \end{aligned} \tag{16}$$

where $\gamma^* = \gamma t$.

Eq. (16) is used to compute the dimensionless temperature profiles inside the substrate material. The laser pulse and material properties used in the computations are given in Table 1.

3. Numerical method

A numerical method (finite difference scheme) is employed to discretize the conduction equation (Eq. (1)). An explicit scheme is used in the numerical method, which is well established in the literature. The stability criteria for the equation discretized is as follows:

$$\left|\frac{2\alpha^2\delta^2}{\Delta\tau}\right| > \left|\frac{\alpha^2\delta^2}{\Delta\tau} - \frac{2k\delta^2}{(\Delta x)^2}\right| + \left|\frac{k\delta^2}{(\Delta x)^2}\right|.$$

The boundary conditions (Eq. (4)) and initial condition (Eq. (5)) are imposed in the numerical simulation. The computations are carried out according to substrate material and pulse properties given in Table 1.

Table 1
Pulse and thermal properties of substrate used in the computation

I_1 (W/m^2)	β and γ (1/s)	δ (1/m)	α (m^2/s)	C_p (J/kg K)	k (W/m K)	ρ (kg/m^3)
2×10^{13}	10^7–10^9	6.16×10^7	0.224×10^{-5}	460	80.3	7880

4. Results and discussions

An analytical solution for the temperature rise due to a time exponentially varying laser heating pulse is obtained using a Laplace transformation method. The closed form solution is simulated for steel. The properties of the substrate material and laser pulse are given in Table 1.

The closed form solution becomes identical to the solution obtained from the previous study [8], once the pulse parameter β is set to zero in Eq. (14), i.e., the closed form solution reduces to a step input intensity pulse. The relevant analysis is given in Appendix A.

Fig. 2(a) shows dimensionless temperature profiles inside the substrate material at different heating periods for the pulse parameter $\beta^*/\gamma^* = 1/3$. Temperature profiles decay gradually in the surface region and sharp decay occurs as the distance from the surface towards the solid bulk increases. This situation is more pronounced at heating period of 1.68. This is because of the pulse profile, i.e. the power intensity is high at the heating period of 1.68 (Fig. 1). Moreover, the gradual decay of temperature profile in the surface vicinity is due to internal energy gain of the substrate material. In this case, energy absorbed by the substrate material from the irradiated field is converted into internal energy gain of the substrate material. Since the power intensity absorbed by the substrate material varies exponentially with depth, temperature rise due to energy gain shows similar trend with the absorbed energy. Consequently, temperature gradient becomes low in the surface vicinity. This can also be seen from Fig. 2(b), in which temperature gradient is shown. Moreover, due to low temperature gradient, the diffusional energy transport from surface vicinity to the solid bulk is small. As the heating period progresses, energy absorbed in the region irradiated by a laser beam results in increase in internal energy gain of the substrate material. Therefore, temperature differential across the surface vicinity and the region next to surface vicinity increases. The diffusional energy transport increases from surface vicinity to the bulk of the substrate material. A stage is reached such that temperature gradient becomes minimum at some depth below the surface. In this case, an energy balance occurs among the absorbed energy, internal energy gain, and diffusional energy transport. The location of minimum temperature gradient changes with the heating period, i.e. depending on the amount of energy absorbed and the heating period, the location of minimum temperature gradient moves away from the surface.

Fig. 3 shows dimensionless temperature profiles inside the substrate material at different heating periods for pulse parameter $\beta^*/\gamma^* = 1/5$. The behavior of temperature curves is similar to those shown in Fig. 2(a), provided that the magnitude of temperature rise is higher for $\beta^*/\gamma^* = 1/3$. This is because of the pulse intensity distribution, i.e., as β^*/γ^* increases the magnitude of peak power intensity increases and the peak power intensity moves towards the pulse beginning (Fig. 1). Consequently, the amount of irradiated energy absorbed by the substrate material reduces during the early heating period for $\beta^*/\gamma^* = 1/5$ pulse profile, which in turn results in less temperature rise inside the substrate material as compared to that corresponding to $\beta^*/\gamma^* = 1/3$.

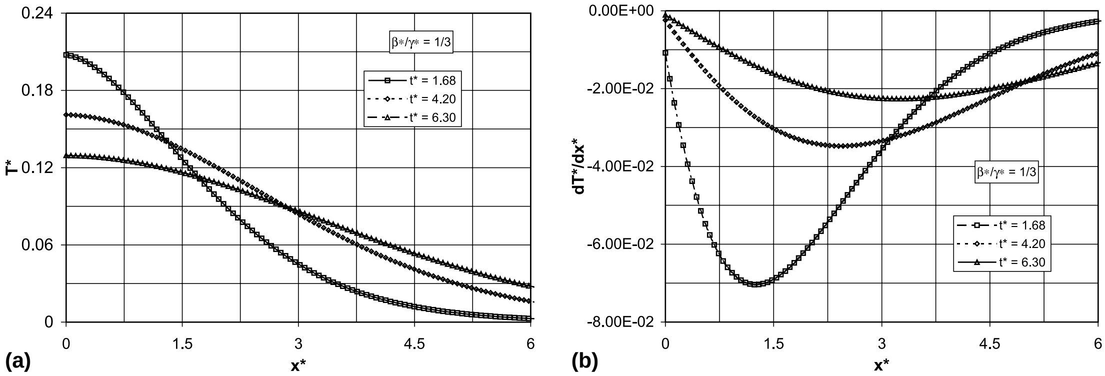

Fig. 2. (a) Dimensionless temperature inside the substrate material at different heating periods for $\beta^*/\gamma^* = 1/3$. (b) Dimensionless temperature gradient inside the substrate material at different heating periods for $\beta^*/\gamma^* = 1/3$.

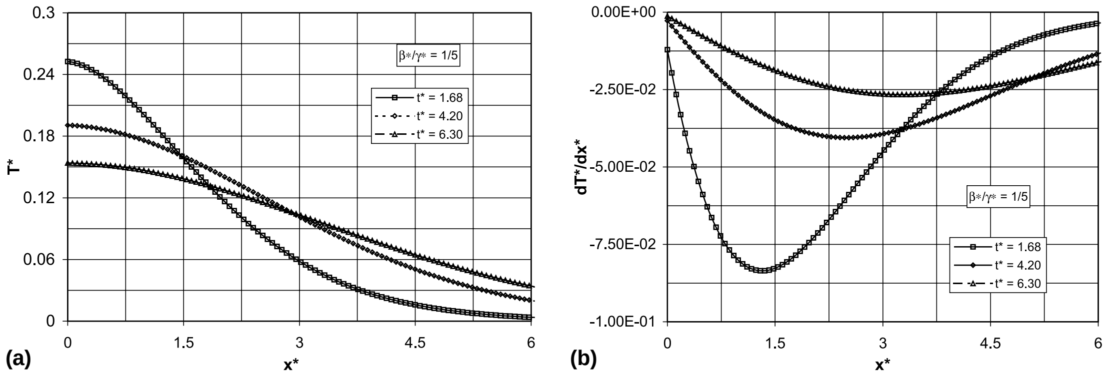

Fig. 3. (a) Dimensionless temperature inside the substrate material at different heating periods for $\beta^*/\gamma^* = 1/5$. (b) Dimensionless temperature gradient inside the substrate material at different heating periods for $\beta^*/\gamma^* = 1/5$.

Fig. 4(a) shows temporal variation of dimensionless temperature at different locations inside the substrate material for pulse parameter $\beta^*/\gamma^* = 1/3$. Temperature profiles predicted from the numerical simulation for $x^* = 0$ and $x^* = 1.2$ are also shown in figure. When comparing temperature profiles obtained from the closed solution with its counterpart predicted from the numerical simulations, both results are in good agreement. The small discrepancies between the both results during the temperature rise period are negligibly small. The rise of temperature in the surface vicinity is higher than those corresponding to some depth below the surface. Moreover, temperature rises rapidly in the early heating period and as the heating period progresses the rate of temperature rise becomes almost steady up to the point of its maxima. This can also be seen from Fig. 4(b), in which time derivative of temperature is shown. The high rate of temperature rise in the surface vicinity is because of the absorption of irradiated laser energy within the absorption depth. In this case, during the early heating period, internal energy gain of the substrate material increases rapidly. This, in turn, results in rapid rise of the temperature in this region. Moreover, the rise of temperature at some depth below the surface ($x^* > 3$) is not considerable in the early heating period. This is because of the energy transport mechanism. Since the amount of absorbed energy is almost negligible in this region (it is below the absorption depth), the rise of temperature is due to diffusional energy transport in this region. In this case, the rate of diffusional energy transport in the early heating period is low due to low temperature gradient in this region. As the heating period progresses, the diffusional energy transport from surface vicinity to solid bulk suppresses the high rate of temperature rise in the surface vicinity. Consequently, temperature rise becomes almost steady with increasing time. At the point of minimum time derivative of temperature ($\partial T^*/\partial t^*$) (Fig. 4(b)), energy balance attains among the absorbed energy, internal energy gain and diffusional energy transport. The time corresponding to minimum $\partial T^*/\partial t^*$ progresses further away from the pulse beginning as the depth from the surface increases towards the solid bulk.

Fig. 5 shows the location of time corresponding to the point of minimum ($\partial T^*/\partial t^*$) for two different pulse parameters. The location of minimum ($\partial T^*/\partial t^*$) moves

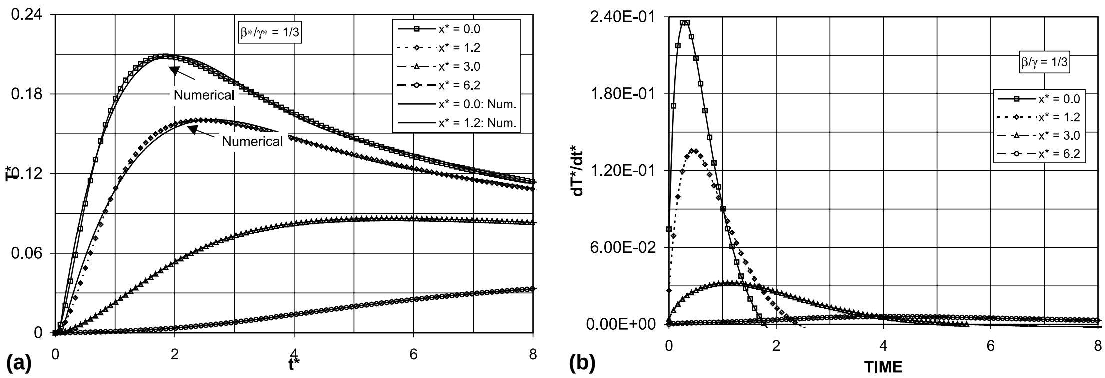

Fig. 4. (a) Dimensionless surface temperature at different location inside the substrate material for $\beta^*/\gamma^* = 1/3$. (b) Time gradient of dimensionless surface temperature at different locations inside the substrate material for $\beta^*/\gamma^* = 1/3$.

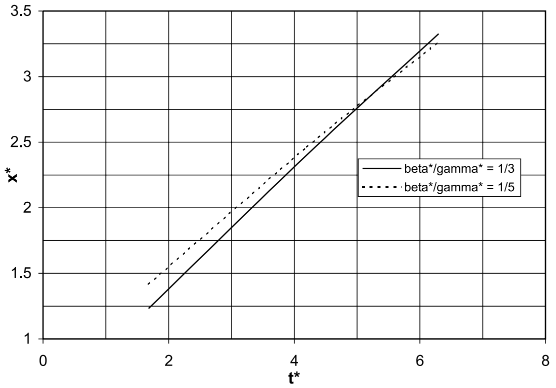

Fig. 5. Dimensionless distance and time corresponding to minimum dT^*/dt^* for two β^*/γ^*.

away from the surface as the heating period progresses. The variation of heating period with depth corresponding to minimum $(\partial T^*/\partial t^*)$ varies linearly. This indicates that the occurrence of steady temperature rise inside the substrate material depends on the heating period, i.e., the steady temperature rise inside substrate material varies at different depths. In this case, increasing depth delays the initiation of steady temperature rise inside the substrate material. It should be noted that the steady temperature rise ends immediately as the rise of temperature diminishes due to the exponential pulse profile. As laser pulse intensity reduces from its peak, temperature decreases from its peak due to time exponentially varying pulse profile. The effect of β^*/γ^* on the location and heating period corresponding to minimum $(\partial T^*/\partial t^*)$ is notable, since the slope of time–distance curve changes in Fig. 5, i.e., reducing β^*/γ^* increases the location of minimum $(\partial T^*/\partial t^*)$ inside the substrate material for a given heating period. This situation reverses as the heating period progresses further ($t^* > 5$).

5. Conclusions

An analytical solution for temperature rise inside a solid substrate due to time exponentially varying laser pulse is presented. The closed form solution is computed for steel to obtain temperature profiles. To examine the effect of pulse parameter (β^*/γ^*) on the resulting temperature profiles, two values of pulse parameter are considered. To validate the closed form solution obtained from the present study, pulse parameter (β) is set to zero in Eq. (14), which in turn results in a closed form solution for a step input intensity pulse as derived from the previous study. It is found that the closed form solution becomes identical to previously obtained analytical solution once β in Eq. (14) is set to zero. Temperatures obtained from the closed form solution are in good agreement with its counterpart predicted from the numerical simulations. Temperature rises rapidly in the surface vicinity of the substrate material in the early heating period and the effect of β^*/γ^* on temperature rise is significant. The specific conclusions derived from the present study can be listed as follows:

1. Temperature rise is higher in the surface vicinity of the substrate material as compared to some depth below the surface. This is because of the absorption of the irradiated energy within the absorption depth. Temperature gradient is lower in the surface vicinity, which indicates that the internal energy gain dominates the diffusional energy transport from surface vicinity to the solid bulk. Moreover, as the depth below the surface increases towards the solid bulk, the diffusional energy transport dominates over the energy exchange mechanism, i.e. internal energy gain of the substrate material due to absorption of the irradiated field becomes negligible beyond the absorption depth.
2. In the early heating period, temperature rises rapidly in the surface vicinity and as the heating progresses, the rise of temperature attains almost steady until it reduces from its maxima. At the point of initiation of steady temperature rise, internal energy gain and diffusional energy transport are in balance such that internal energy rises steadily.
3. The time and location corresponding to $(\partial T^*/\partial t^*)$ maxima varies linearly. In this case, increasing time results in increasing location of $(\partial T^*/\partial t^*)$ maxima inside the substrate material. Moreover, the effect of pulse parameter (β^*/γ^*) on the resulting $(\partial T^*/\partial t^*)$ maxima is significant. The location of $(\partial T^*/\partial t^*)$ maxima occurs in the early heating period as β^*/γ^* increases.

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Appendix A. Closed from solution for conduction limited heating case

The Fourier heat transfer equation governing the laser step input pulse intensity can be written as

$$k\frac{\partial^2 T}{\partial x^2} + I_0(1 - r_f)\delta\exp(-\delta x) = \rho C_p\frac{\partial T}{\partial t} \qquad (A.1)$$

with the initial condition

At time $t = 0 \rightarrow T(x, 0) = 0$.

The boundary conditions:

At the surface $\Rightarrow \quad x = 0 \rightarrow \left.\frac{\partial T}{\partial x}\right|_{x=0} = 0$

and

x at infinity $\Rightarrow \quad x = \infty \rightarrow T(\infty, t) = 0.$

The solution of Eq. (A.1) is obtained in the previous study using a Laplace transformation method [10], which yields

$$T(x,t) = \frac{2I_0(1-r_f)}{k}\sqrt{\alpha t}\,\mathrm{i\,erf}\,c\left(\frac{x}{2\sqrt{\alpha t}}\right) - \frac{I_0}{k\delta} \times \exp(-\delta x) + \frac{I_0}{2k\delta}\exp(\alpha\delta^2 t - \delta x) \times \mathrm{erf}\,c\left(\delta\sqrt{\alpha t} - \frac{x}{2\sqrt{\alpha t}}\right) + \frac{I_0}{2k\delta} \times \exp(\alpha\delta^2 t + \delta x)\mathrm{erf}\,c\left(\delta\sqrt{\alpha t} + \frac{x}{2\sqrt{\alpha t}}\right), \quad \text{(A.2)}$$

where

$$\mathrm{i\,erf}\,c(z) = \frac{1}{\sqrt{\pi}}\exp(-z^2) - z\,\mathrm{erf}\,c(z).$$

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